Mathematical Modeling Issues in the Future Wireless Networks

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- Loss systems
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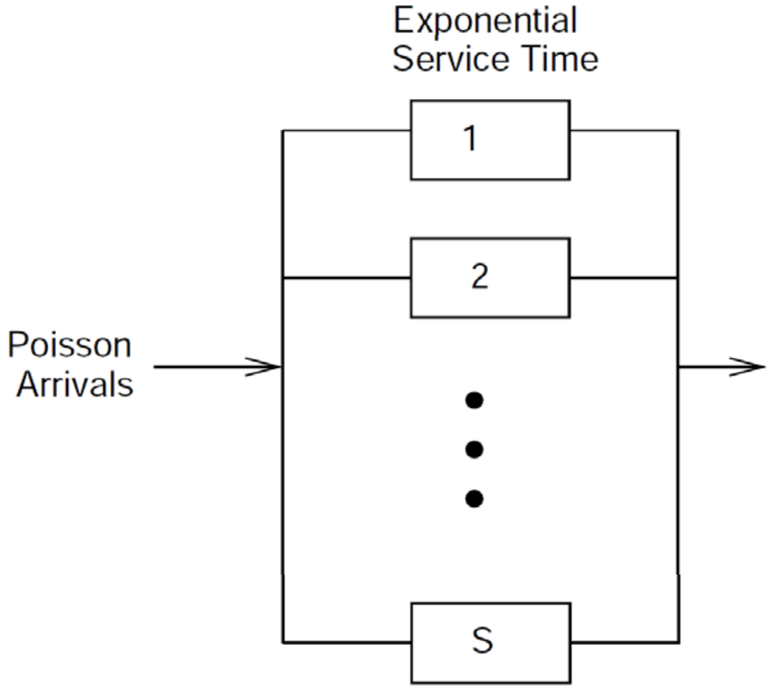
Loss systems

- Agner Krarup Erlang (1878 - 1929)
  "*Solution of some Problems in the Theory of Probabilities of Significance in Automatic Telephone Exchanges*" (1917).
- If in an average ρ customers arrive during mean occupation time, the stationary distribution is given by



$$p_n = \frac{\frac{\rho^n}{n!}}{\sum_{v=0}^{S} \frac{\rho^v}{v!}}, \quad 0 \leq n \leq S$$

Generalized loss systems

- *Multi-class sources:* Class k customers arrive as a Poisson process with rate λ_k with the mean holding time $1/\mu_c$
- *Simultaneous acquisition of multiple servers:* A class k customer requires to hold s_k servers simultaneously.
- The set of feasible states is

$$\mathcal{X} = \{\mathbf{n} \in \mathbb{N}^K \mid \sum_{k=1}^{K} s_k n_k \le S\}$$

- The stationary distribution is given by

$$p_{\mathbf{n}} = \frac{1}{G} \prod_{k=1}^{K} \frac{\rho_k^{n_k}}{n_k!}, \quad \mathbf{n} \in \mathcal{X}, \qquad G = \sum_{\mathbf{n} \in \mathcal{X}} \prod_{k=1}^{K} \frac{\rho_k^{n_k}}{n_k!},$$

where

$$\rho_k = \lambda_k / \mu_k$$

Loss networks

- Simultaneous acquisition of multiple *servers of different types:* There are S_m servers of type m. A class k customers requires to s_{km} hold servers of type m simultaneously.
- The preceding formulas for the stationary distribution are valid. Only the set of feasible states is different

$$\mathcal{X} = \{\mathbf{n} \in \mathbb{N}^K \mid \sum_{k=1}^{K} s_{km} n_k \leq S_m, m = 1,2,\ldots M\}$$

- **The loss network provides a general model for a circuit-switched network that carries multi-rate traffic**
- The model is equally applicable to bidirectional flows.
- The reverse traffic for a given pair of nodes may have different bandwidth requirements

Loss systems with random resource demands

- Acquisition of multiple *resources of different types:*
  - There are R_m units of resources of type *m*, $\mathbf{R} = (R_1, \dots, R_M)$
  - The *i*-th customer of class *k* requires to hold $r_{mk}(i)$ units of resources of type *m*.
  - Resource demands $\mathbf{r}_k(i) = (r_{k1}(i), \dots, r_{kM}(i)),\ i = 1, 2, \dots$ of class *k* customers are nonnegative random vectors with cumulative distribution functions $F_k(\mathbf{x})$
- The set of feasible states is given by

$$\mathcal{X} = \{(\mathbf{n}, \boldsymbol{\gamma}_1, \dots, \boldsymbol{\gamma}_K) \mid \mathbf{n} \in \mathbb{N}^K, \boldsymbol{\gamma}_k \in \mathbb{R}_+^M, k = 1, 2, \dots, K, \\ \sum_{k=1}^{K} \boldsymbol{\gamma}_k \le \mathbf{R}, \sum_{k=1}^{K} n_k \le S\}$$

$\mathbf{n} = (n_1, \dots, n_K)$ – population vector

$\boldsymbol{\gamma}_k = (\gamma_{k1}, \dots, \gamma_{kM})$ – vector of resources occupied by class *k* customers

Loss systems with random resource demands (continued)

- Cumulative distribution functions of the stationary distribution are given by

$$P_{\mathbf{n}}(\mathbf{x}_1,\ldots,\mathbf{x}_K) = \frac{1}{G}\prod_{k=1}^{K}\frac{\rho_k^{n_k}}{n_k!}F_k^{*n_k}(\mathbf{x}_k), \quad (\mathbf{n},\mathbf{x}_1,\ldots,\mathbf{x}_K) \in \mathcal{X},$$

$$G = \sum_{n_1+\ldots+n_K \le S}(F_1^{*n_1} * \ldots * F_K^{*n_K})(\mathbf{R})\frac{\rho_1^{n_1}\cdots\rho_K^{n_K}}{n_1!\cdots n_K!}$$

$*$ – convolution symbol

- Blocking probability of class k customers: $B_k = 1 - \dfrac{G_k}{G}$,

$$G_k = \sum_{n_1+\ldots+n_K < S}(F_1^{*n_1} * \ldots * F_k^{*(n_k+1)} * \ldots * F_K^{*n_K})(\mathbf{R})\frac{\rho_1^{n_1}\cdots\rho_K^{n_K}}{n_1!\cdots n_K!}$$

Loss systems with positive and negative resource demands

- Resource demands $\mathbf{r}_k(i) = (r_{k1}(i),\ldots,r_{kM}(i)),\ \ i = 1,2,\ldots$ of class k customers are random ~~nonnegative~~ vectors with cumulative distribution function $F_k(\mathbf{x})$
- Acquisition of a *positive* quantity of a resource means *subtraction* of this quantity from the pool of available resources
- Acquisition of a *negative* quantity of a resource means *addition* of this quantity to the pool of available resources
- A customer with negative resource demand can leave the system only if the resource that was added to the pool of available resources can be picked up without disrupting the service of other calls.

Loss systems with positive and negative resource demands (cont)

- The preceding formulas for the stationary distribution are valid, but there is a slight difference between the sets of feasible states

$$\mathcal{X} = \{(\mathbf{n}, \boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_K) \mid \mathbf{n} \in \mathbb{N}^K, \boldsymbol{\gamma}_k \in \mathbb{R}^M, k = 1, 2, \ldots, K,$$
$$\sum_{k=1}^{K} \boldsymbol{\gamma}_k \le \mathbf{R}, \sum_{k=1}^{K} n_k \le N\}$$

- Particular case: loss systems with positive and negative customers
  - Two groups of customers.
  - Customers from the first group always acquire only positive quantities of resources.
  - Customers from the second group always acquire only negative quantities of resources

Loss networks with random resource demands and signals

- Network contains customers and signals
- Arriving signal interrupts the service of a customer and forces a customer to leave the network, or to move instantaneously to another loss system where the customer requests new service.
- If the service of a customer was not interrupted, the customer leaves the nework and is considered as successfully served
- Important characteristic are the probability that a class k customer, initially arriving at node i, is served successfully, the mean number of interruptions etc.

Thank You!